hep-lat/9610005
LBNL-39424
SWAT-96-119
September 1996



# Center Dominance and $Z_2$ Vortices In $SU(2)$ Lattice Gauge Theory

*L. Del Debbio*[a], *M. Faber*[b], *J. Greensite*[c,d], *and Š. Olejník*[e]

[a] Department of Physics, University of Wales Swansea,
SA2 8PP Swansea, UK. E-mail: `L.Del-Debbio@swansea.ac.uk`

[b] Institut für Kernphysik, Technische Universität Wien,
1040 Vienna, Austria. E-mail: `faber@kph.tuwien.ac.at`

[c] Physics and Astronomy Dept., San Francisco State University,
San Francisco CA 94132 USA. E-mail: `greensit@stars.sfsu.edu`

[d] Theoretical Physics Group, Ernest Orlando Lawrence Berkeley
National Laboratory, University of California, Berkeley,
California 94720 USA. E-mail: `greensite@theorm.lbl.gov`

[e] Institute of Physics, Slovak Academy of Sciences,
842 28 Bratislava, Slovakia. E-mail: `fyziolej@savba.savba.sk`

## Abstract

We find, in close analogy to abelian dominance in maximal abelian gauge, the phenomenon of center dominance in maximal center gauge for $SU(2)$ lattice gauge theory. Maximal center gauge is a gauge-fixing condition that preserves a residual $Z_2$ gauge symmetry; "center projection" is the projection of $SU(2)$ link variables onto $Z_2$ center elements, and "center dominance" is the fact that the center-projected link elements carry most of the information about the string tension of the full theory. We present numerical evidence that the thin $Z_2$ vortices of the projected configurations are associated with "thick" $Z_2$ vortices in the unprojected configurations. The evidence also suggests that the thick $Z_2$ vortices may play a significant role in the confinement process.

# 1 Introduction

Perhaps the most popular theory of quark confinement is the dual-superconductor picture as formulated in abelian-projection gauges [1]. In this theory the unbroken $U(1)^{N-1}$ symmetry of the full $SU(N)$ gauge group plays a special role in identifying both the relevant magnetic monopole configurations, and also the abelian charge which is subject to the confining force. The phenomenon of abelian dominance [2] in maximal abelian gauge [3] is often cited as strong evidence in favor of the dual-superconductor picture (c.f. ref. [4] for a recent review).

Of course, many alternative explanations of quark confinement have been advanced over the years. The theory which will concern us in this article is the vortex condensation (or "spaghetti vacuum") picture, in which the vacuum is understood to be a condensate of vortices of some finite thickness, carrying flux in the center of the gauge group. The spaghetti picture was originally advanced by Nielsen and Olesen [5], and this idea was further elaborated by the Copenhagen group in the late seventies. A closely related idea, due to 't Hooft [6] and Mack [7], emphasized the importance of the $Z_N$ center of the $SU(N)$ gauge group. In that picture there is a certain correspondence between magnetic flux of the relevant vortices and the elements of the the $Z_N$ subgroup, and it is random fluctuations in the number of such vortices linked to a Wilson loop which explains the area-law falloff.[1] Ref. [10] presents an argument for this $Z_N$ center restriction in the framework of the "Copenhagen vacuum."

The vortex condensation theory, like dual-superconductivity, focuses on on a certain subgroup of the full $SU(N)$ gauge group, but it is the $Z_N$ center, rather than $U(1)^{N-1}$, which is considered to be of special importance. This raises a natural question: Does there exist, in close analogy to abelian dominance, some version of "center dominance?" If so, should this evidence be interpreted essentially as a critique of abelian dominance, or should it be viewed as genuine support for the vortex theory? Supposing that the vortex condensation theory is taken seriously, how can one identify $Z_2$ vortices in unprojected field configurations, and can one determine if such vortices are of any physical importance? This article is intended as a preliminary investigation of these questions.

[1]See also ref. [8]. Recent work along these same lines is found in ref. [9].

## 2 Center Dominance

We begin with the phenomenon of "center dominance" in maximal center gauge. One starts by fixing to the maximal abelian gauge [3], which, for $SU(2)$ gauge theory, maximizes the quantity

$$\sum_x \sum_{\mu=1}^{4} \mathrm{Tr}[\sigma_3 U_\mu(x) \sigma_3 U_\mu^\dagger(x)] \tag{1}$$

This gauge has the effect of making link variables as diagonal as possible, leaving a remnant $U(1)$ gauge symmetry. "Abelian projection" means the replacement of the full link variables $U$ by the abelian links $A$, according to the rule

$$U = a_0 I + i\vec{a}\cdot\vec{\sigma} \quad \longrightarrow \quad A = \frac{a_0 I + i a_3 \sigma^3}{\sqrt{a_0^2 + a_3^2}} \tag{2}$$

It can be shown that the $A$ link variables transform like $U(1)$ gauge fields under the remnant $U(1)$ symmetry. Abelian dominance, found by Suzuki and collaborators [2], is essentially the fact that the confining string tension can be extracted from the abelian-projected $A$-link variables alone. Abelian dominance has been widely interpreted as supporting the dual-superconductor theory advanced in ref. [1].

But while the dual-superconductor idea focuses on the remnant $U(1)$ subgroup of the gauge symmetry, it is the $Z_2$ center of the $SU(2)$ gauge group that seems most relevant in the vortex condensation picture. This suggests making a further gauge-fixing, which would bring the abelian links as close as possible to the center elements $\pm I$ of $SU(2)$. Therefore, writing

$$A = \begin{bmatrix} e^{i\theta} & \\ & e^{-i\theta} \end{bmatrix} \tag{3}$$

we use the remnant $U(1)$ symmetry to maximize

$$\sum_x \sum_\mu \cos^2(\theta_\mu(x)) \tag{4}$$

leaving a remnant $Z_2$ symmetry. This we call "Maximal Center Gauge." Then define, at each link,

$$Z \equiv \mathrm{sign}(\cos\theta) = \pm 1 \tag{5}$$

which transforms like a $Z_2$ gauge field under the remnant symmetry. "Center Projection" $U \to Z$, analogous to "abelian projection" $U \to A$, is defined as the replacement of the full link variables $U$ by the center element $ZI$, in the computation of observables such as Wilson loops and Polyakov lines.

Figure 1 is a plot of Creutz ratios vs. coupling $\beta$, extracted from Wilson loops formed from the center-projected $Z_2$ link variables. Lattice sizes were $10^4$ for $\beta \leq 2.3$, $12^4$ at $\beta = 2.4$, and $16^4$ at $\beta = 2.5$.[2] What is rather striking about Fig. 1 is the fact that, from 2 lattice spacings onwards, the Creutz ratios at fixed $\beta \geq 2.1$ all fall on top of one another, and all lie on the same scaling line

$$\sigma a^2 = \frac{\sigma}{\Lambda^2}(\frac{6}{11}\pi^2\beta)^{102/121} \exp[-\frac{6}{11}\pi^2\beta] \tag{6}$$

with the value $\sqrt{\sigma}/\Lambda = 67$. Even the logarithm of the one-plaquette loop, $\chi(1,1)$, appears to parallel this line. This behavior is in sharp contrast to Creutz ratios extracted from the full link variables, where only the envelope of Creutz ratios fits the scaling line.

The equality of Creutz ratios, starting at 2 lattice spacings, means that the center projection sweeps away the short-distance, $1/r$-type potential, and the remaining linear potential is revealed already at short distances. This fact is quite apparent in Fig. 2, which displays the data for $\chi(R,R)$ at $\beta = 2.4$ for the full theory (crosses), the center projection (diamonds), and also for the $U(1)/Z_2$-projection (squares). The latter projection consists of the replacement $U \to A/Z$ for the link variables. We note that the center-projected data is virtually flat, from $R = 2$ to $R = 5$, which means that the potential is linear in this region, and appears to be the asymptote of the full theory. It should also be noted that abelian link variables with the center factored out, i.e. $U \to A/Z$, appear to carry no string tension at all.

Of course, one can also carry out finite temperature studies in the center projection. Thus far, we have only computed Polyakov lines vs. $\beta$ on a $6^3 \times 2$ lattice, and obtained the results shown in Fig. 3. The deconfinement transition, signaled by a sudden jump in the value of the Polyakov line, appears to occur at the value of $\beta$ appropriate for $T = 2$ lattice spacings in the time direction.

It should be noted parenthetically that our definition of maximal center gauge is not the only possible definition. A similar but not identical gauge, leaving a remnant $Z_2$ symmetry, would be the gauge which maximizes

$$\sum_x \sum_\mu \{\mathrm{Tr} U_\mu(x)\}^2 \tag{7}$$

[2]Finite size effects, as indicated by the values of center-projected Polyakov lines, appear to be significantly larger for center-projected configurations as compared to the full link variables, and this is why we use a $16^4$ lattice at $\beta = 2.5$.

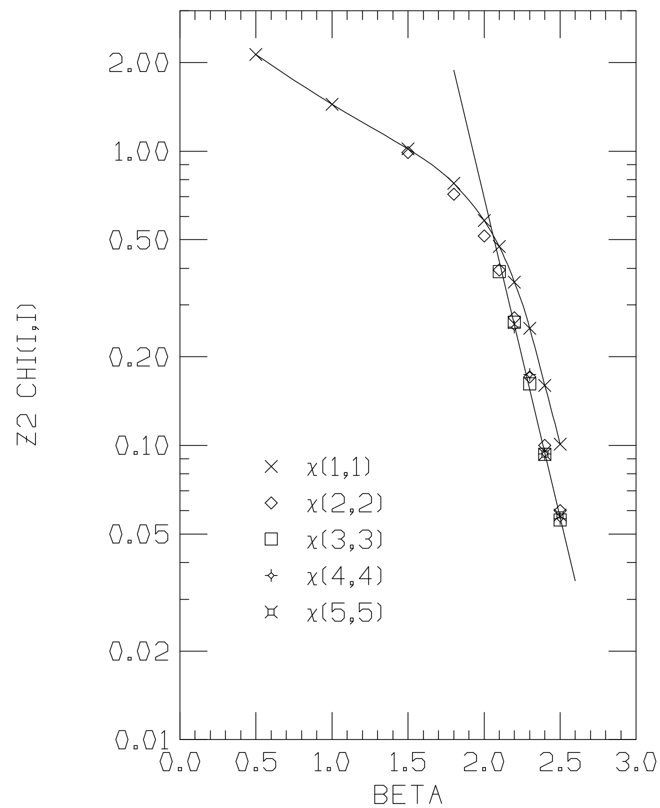


Figure 1: Creutz ratios from center-projected lattice configurations.

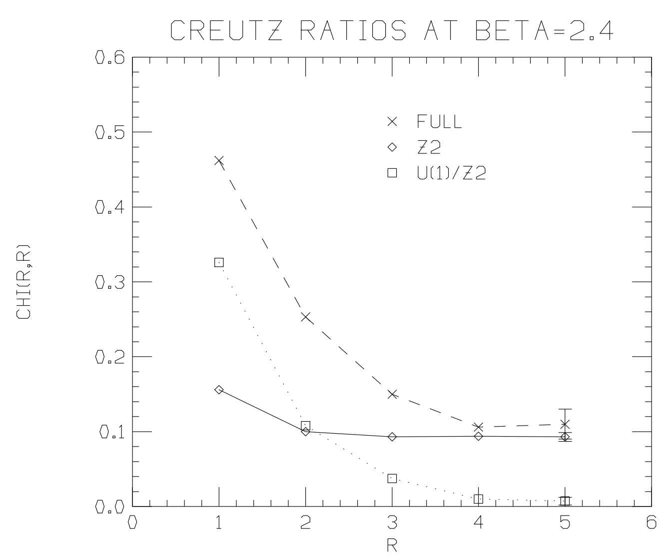


Figure 2: Creutz ratios $\chi(R,R)$ vs. $R$ at $\beta = 2.4$, for full, center-projected, and $U(1)/Z_2$-projected lattice configurations.

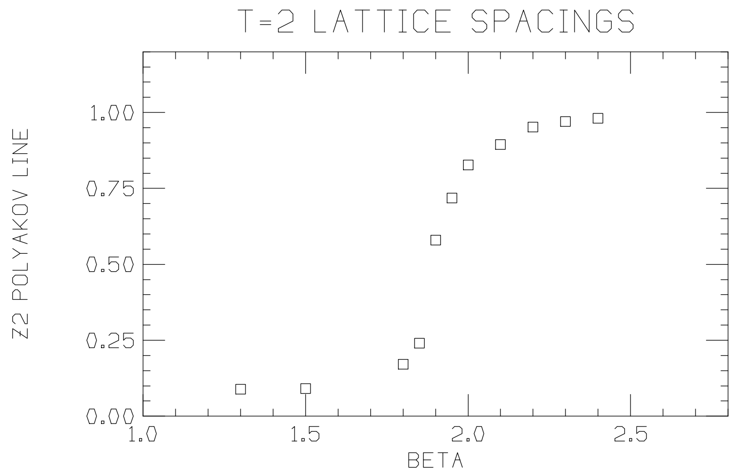


Figure 3: Polyakov lines vs. $\beta$ in center projection, for a $6^3 \times 2$ lattice.

with center projection defined by

$$Z = \text{sign}(\text{Tr}U) \tag{8}$$

This version is more difficult to implement numerically, and has not yet been studied.

In any case, center dominance in maximal center gauge, as displayed in the figures above, does not necessarily imply that confinement is due to vortex condensation. In fact our initial view, expounded in ref. [11], was that since center dominance would appear to support a theory, namely vortex condensation, which is "obviously wrong" (for reasons discussed in section 4), its success only proves that neither center dominance nor abelian dominance are reliable indicators of the confinement mechanism. The truncation of degrees of freedom inherent in both the abelian and the center projections may easily do violence to the topology of the confining gauge fields. So the fact that the confining configurations of $U(1)$ gauge fields are monopoles, while confining configurations in $Z_2$ gauge theory are condensed vortices, does not necessarily imply that either type of configuration is especially relevant to the full, unprojected $SU(2)$ theory.

However, before stating with assurance that the $Z_2$ vortices of the center-projected configurations have *nothing* to do with confinement, there are certain checks that must be carried out. It is here that we have encountered a surprise.

## 3 The Detection of $Z_2$ Vortices

Consider a field configuration $U_\mu(x)$, and any planar loop $C$. As explained above, it is a simple matter to transform to maximal center gauge, and then to examine each of the plaquettes spanning the minimal area enclosed by loop $C$, in the corresponding center projected configuration $Z_\mu(x)$. The number of plaquettes computed with center-projected links, whose value is $-1$, corresponds to the number of $Z_2$ vortex lines of the center-projected configuration which pierce the minimal loop area. We will refer to these $Z_2$ vortex lines of the center-projected configurations as "projection-vortices," or just "P-vortices," to distinguish them from the (hypothetical) $Z_2$ vortices that might be present in the unprojected configurations. As a Monte Carlo simulation proceeds, the number of P-vortices piercing any given loop area will fluctuate. The first question to ask is whether the presence or absence of P-vortices in the projected configurations is correlated in any way with the confining properties of the corresponding unprojected configurations.

To answer this question, we compute Creutz ratios $\chi_0(R,R)$ of Wilson loops $W_0(C)$, that are evaluated in a subensemble of Monte Carlo-generated configurations in which no P-vortex pierces the minimal area of loop $C$. We stress that the full, unprojected link variables are used in computing the loop, and the center-projection is employed only to select the data set. In practice, having generated a lattice configuration and fixed to maximal center gauge, one examines each rectangular loop of a given size; those with no P-vortices piercing the loop are evaluated, and those with a non-zero number are skipped. Of course, by a trivial generalization, we may compute Wilson loops $W_n(C)$, evaluated in ensembles of configurations with any given number $n$ of P-vortices piercing the loop.

Figure 4 displays Creutz ratios $\chi_0(R,R)$ extracted from $W_0(C)$ loops, as compared to the standard Creutz ratios with no such restriction, at $\beta = 2.3$. From this figure it is clear that, while the zero-vortex restriction makes little difference to the smallest loops, it makes a very big difference to the Creutz ratios of the larger loops. It appears, in fact, that the aymptotic string tension of the zero-vortex loops vanishes altogether.[3]

If we presume (as most people do) that confinement is an effect associated with some particular type of field configuration - let us call them the "Confiners" - then it would seem from Fig. 4 that the presence or absence of P-vortices in the center-

[3]Error bars are much smaller for the no-vortex data as compared to the full data; this is why we can report meaningful results at larger $R$ for the no-vortex data than for the full data.

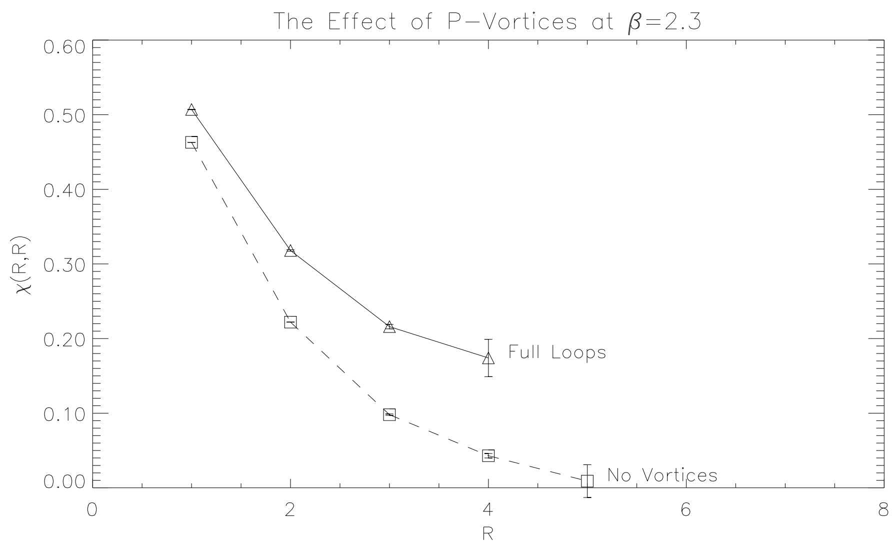


Figure 4: Creutz ratios $\chi_0(R,R)$ extracted from loops with no P-vortices, as compared to the usual Creutz ratios $\chi(R,R)$, at $\beta = 2.3$.

projected configurations is strongly correlated with the presence or absence of Confiners in the unprojected configurations. The next question is whether we can exclude the possibility that these Confiners are actually $Z_2$ vortices.

To address this question, assume for the moment that to each P-vortex piercing a given loop, there corresponds a $Z_2$ vortex in the full, unprojected field configuration piercing that loop. This assumption has the consequence that, in the limit of large loops,

$$\frac{W_n(C)}{W_0(C)} \quad \longrightarrow \quad (-1)^n \tag{9}$$

The argument for eq. (9) goes as follows: In $SU(2)$ (as opposed to $Z_2$) lattice gauge theory, the field strength of a vortex may be spread out in a cross-section, or "core," of some finite diameter $D$ greater than one lattice spacing. Outside the core, the vector potential of each vortex can be represented by a discontinuous gauge transformation. If a surface bounded by loop $C$ is pierced by $n$ vortex lines, and if the cores of the vortices do not intersect $C$, then the relevant gauge transformation, at the point of discontinuity, has the property

$$g(x(0)) = (-1)^n g(x(1)) \tag{10}$$

where $x^\mu(\tau),\ \tau \in [0,1]$ parametrizes the closed loop $C$. We can then decompose the vector potential $A^{(n)}_\mu(x)$ in the neighborhood of loop $C$ in terms of a discontinuous gauge transformation $g(x)$, which represents the vortex background near $C$, and a fluctuation $\delta A^{(n)}_\mu(x)$ such that

$$A^{(n)}_\mu(x) = g^{-1}\delta A^{(n)}_\mu(x) g + ig^{-1}\partial_\mu g \tag{11}$$

The corresponding Wilson loop, evaluated on the subensemble of configurations in which $n$ vortex lines pierce loop $C$, would be

$$\begin{aligned} W_n(C) &= < \mathrm{Tr}\exp[i\oint dx^\mu A^{(n)}_\mu] > \\ &= (-1)^n < \mathrm{Tr}\exp[i\oint dx^\mu \delta A^{(n)}_\mu] > \end{aligned} \tag{12}$$

Of course, any vector potential in the neighborhood of loop $C$ can be rewritten in the form (11), so given some criterion for identifying the number of $Z_2$ vortex lines piercing loop $C$ (such as counting P-vortices), the question is whether this criterion, and the corresponding decomposition (12), is physically meaningful. A reasonable test is to see if the probability distribution of fluctuations $\delta A^n(x)$ is independent of the number of vortex lines piercing the loop. This test is based on the fact that, in any local region of a large loop $C$, the effect of the vortices is simply a gauge transformation. Thus, providing the fluctuations $\delta A^n(x)$ have only short range correlations, their distribution in the neighbourhood of loop $C$ should be unaffected by the presence or absence of vortex lines in the middle of the loop. Therefore, if we have correctly isolated the vortex contribution,

$$< \mathrm{Tr}\exp[i\oint dx^\mu \delta A^{(n)}_\mu] > \;\approx\; < \mathrm{Tr}\exp[i\oint dx^\mu \delta A^{(0)}_\mu > \tag{13}$$

for sufficiently large loops. This immediately leads to eq. (9); all that is needed is test this equation.

Figure 5 shows the ratio $W_1(C)/W_0(C)$ vs loop area, for rectangular ($R\times R$ and $(R+1)\times R$) loops at $\beta = 2.3$. The simulations were performed on a $14^4$ lattice with 1000 thermalizing sweeps, followed by 8000 sweeps, with data taken every 10th sweep. In order to give the loop $W_1(C)$ the greatest chance to lie outside the vortex core (assuming it exists), $W_1(C)$ was evaluated in the subensemble of configurations in which the single P-vortex is located in the center of the loop. The data seems perfectly consistent with eq. (9), i.e. $W_1(C)/W_0(C) \rightarrow -1$ as the loop area increases.

Figure 6 shows the corresponding ratio $W_2(C)/W_0(C)$ vs loop area. In this case we have evaluated $W_2(C)$ in the subensemble of configurations in which the two P-vortices lie inside a $2\times 2$ square in the middle of the loop. As in the 1-vortex case,

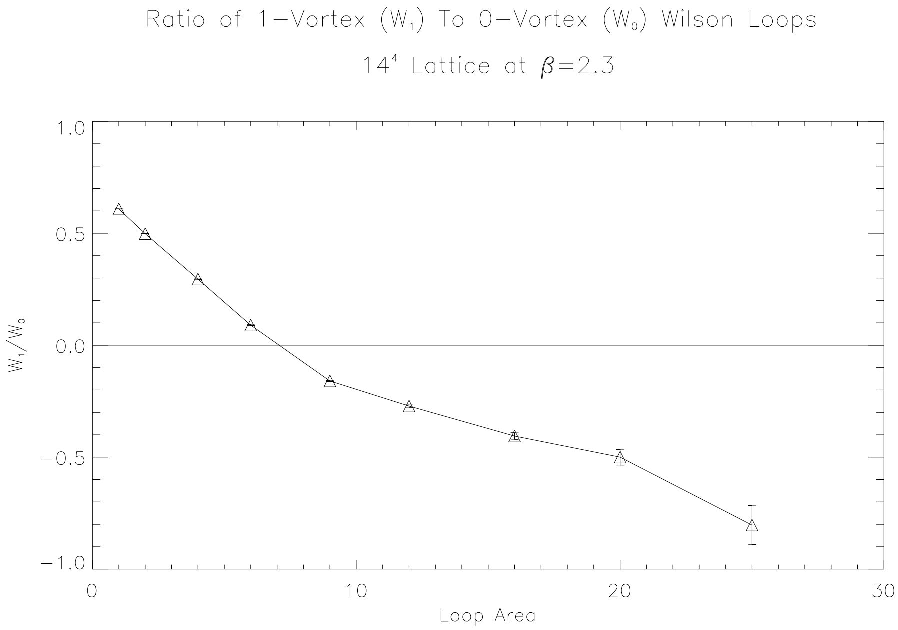


Figure 5: Ratio of the 1-Vortex to the 0-Vortex Wilson loops, $W_1(C)/W_0(C)$, vs. loop area at $\beta = 2.3$.

the idea is to keep the loop $C$ as far as a possible from the vortex cores, although the cores themselves may overlap. Once again, the data seems to agree nicely with eq. (9) for $n = 2$, i.e. $W_2(C)/W_0(C) \to +1$.

Of course, the configurations that contain exactly zero (or exactly one, or two) P-vortices piercing a given loop become an ever smaller fraction of the total number of configurations, as the loop area increases. However, for increasingly large loops, one would expect that the fraction of configurations with an even number of P-vortices piercing the loop, and the fraction with an odd number piercing the loop, approach one another. This is indeed the case, as can be seen from Figure 7.

Let us define $W_{evn}(C)$ to be the Wilson loops evaluated in configurations with an even (including zero) number of P-vortices piercing the loop, and $W_{odd}(C)$ the corresponding quantity for odd numbers.[4] According to eq. (9), $W_{evn}(C)$ and $W_{odd}(C)$ should be of opposite sign, for large loop area. Moreover, according to the vortex condensation picture, the area law for the full loop $W(C)$ is due to fluctuations in the $\pm 1$ factor, coming from fluctations in even/odd numbers of vortices piercing the

[4]In evaluating $W_{evn}$ and $W_{odd}$, we make no special restriction on the location of the P-vortices within the loop.

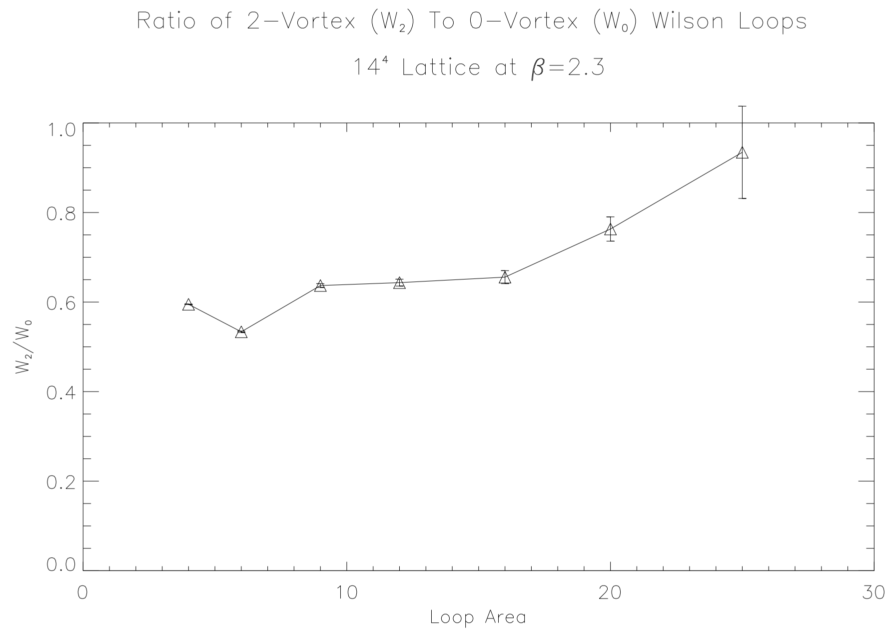


Figure 6: Ratio of the 2-Vortex to the 0-Vortex Wilson loops, $W_2(C)/W_0(C)$, vs. loop area at $\beta = 2.3$.

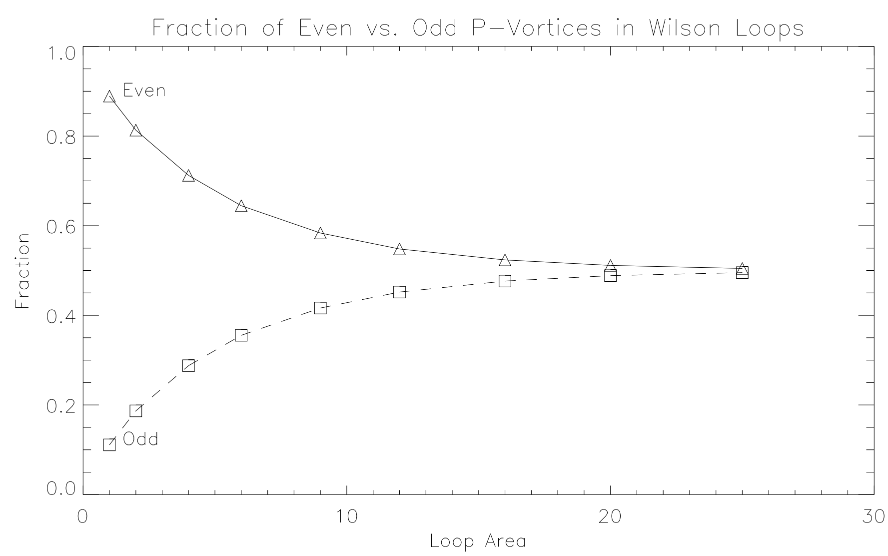


Figure 7: Fraction of link configurations containing even/odd numbers of P-vortices, at $\beta = 2.3$, piercing loops of various areas.

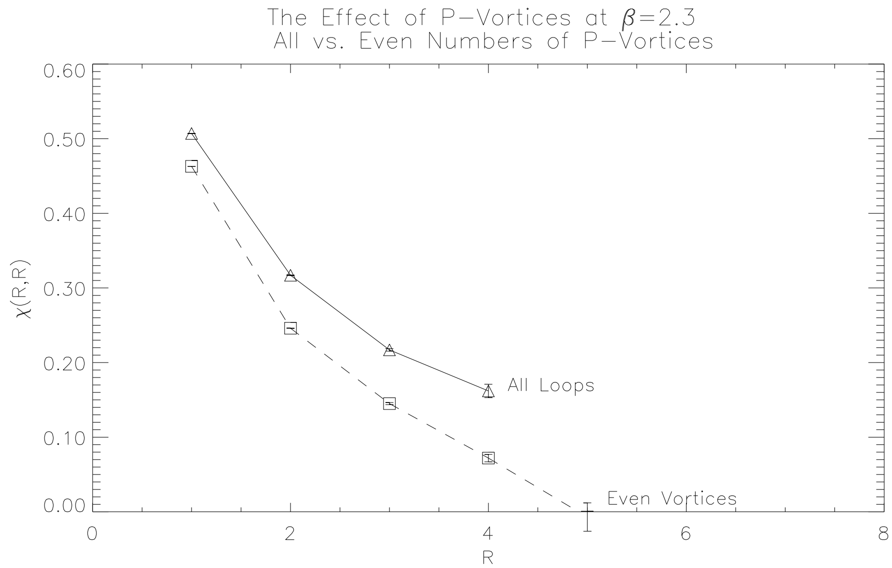


Figure 8: Creutz ratios $\chi_{ev}(R,R)$ extracted from Wilson loops $W_{evn}(C)$, taken from configurations with even numbers of P-vortices piercing the loop. The standard Creutz ratios $\chi(R,R)$ at this coupling ($\beta = 2.3$) are also shown.

loop. If this is the case, then neither $W_{evn}(C)$ alone, nor $W_{odd}(C)$ alone, would have an area law, but only the weighted sum

$$W(C) = P_{evn}(C)W_{evn}(C) + P_{odd}(C)W_{odd}(C) \tag{14}$$

where $P_{evn}$ and $P_{odd}$ are the fractions of configurations, shown in Fig. 7, with even/odd numbers of P-vortices piercing the loop. For large loops, $P_{evn} \approx P_{odd} \approx 0.5$.

Figure 8 shows the Creutz ratios extracted from $W_{evn}(C)$, compared to the standard Creutz ratios at $\beta = 2.3$. The figure is qualitatively quite similar to Fig. 4, but here it should be emphasized that the data set used to evaluate $W_{evn}(C)$ is not a small minority of configurations (as it is for $W_0(C)$ for large loops), but constitutes at least half the configurations. The asympotic string tension, extracted from these configurations, appears to vanish.

Figure 9 shows the values of $W_{evn}(C)$, $W_{odd}(C)$, $W(C)$ vs. loop area, for the larger loops. As expected from eq. (9), $W_{evn}$ and $W_{odd}$ have opposite signs. The full Wilson loop $W(C)$ has a positive sign, but is substantially smaller, at loop area $\geq$ 20, than either of its two components. If this behavior persists at still greater areas,

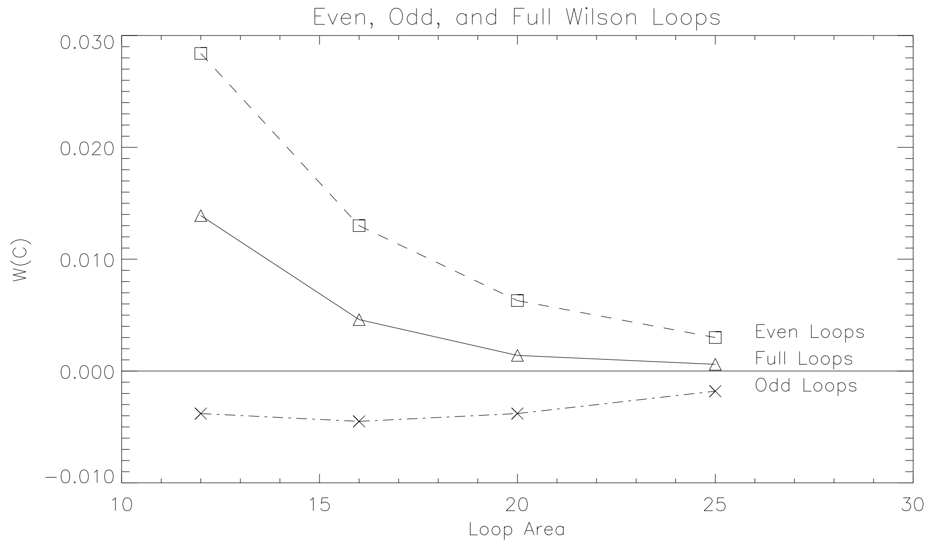


Figure 9: Wilson loops $W_{evn}(C)$, $W_{odd}(C)$ and $W(C)$ at larger loop areas, taken from configurations with even numbers of P-vortices, odd numbers of P-vortices, and any number of P-vortices, respectively piercing the loop. Again $\beta = 2.3$.

then the area law falloff of a Wilson loop $W(C)$ is due to a very delicate cancellation between much larger positive and negative components, associated with even and odd numbers of P-vortices respectively. Neither $W_{evn}(C)$ nor $W_{odd}(C)$, by itself, would appear to have an area law.

# 4 Against Vortices

The data presented in the previous section suggests that $Z_N$ vortices play a crucial role in the confinement process, and that condensation of such vortices (as proposed in refs. [5, 6, 7]) may be the long-sought confinement mechanism. On the other hand, there are some serious objections which can be raised against this mechanism. We have raised these objections repeatedly, in connection with the abelian-projection theory [12, 13, 11], and they apply with even more force to the vortex-condensation theory.

The difficulties are all associated with Wilson loops in higher group representations. First of all, there is a problem concerning the large-N limit [14]. A Wilson loop for quarks in the adjoint representation of an $SU(N)$ gauge group is unaffected by the discontinuous gauge transformations associated with $Z_N$ vortices; it follows that fluctuations in the number of such vortices cannot produce an area law for adjoint loops. On the other hand, it is a consequence of factorization in the large-N limit that, at $N = \infty$, the string tension of the adjoint loop $\sigma_{Adj}$ is simply related to the string tension of the $\sigma_{fund}$ of the fundamental loops

$$\sigma_{Adj} = 2\sigma_{fund} \tag{15}$$

In addition, the existence of an adjoint string tension does not appear to be just a peculiarity of the large-N limit. It has been found in numerous Monte Carlo investigations, for both the $SU(2)$ and $SU(3)$ gauge groups in both three and four dimensions, that there is an intermediate distance regime, from the onset of confinement to the onset of color screening, where

$$\frac{\sigma_r}{\sigma_{fund}} \approx \frac{C_r}{C_{fund}} \tag{16}$$

where $C_r$ is the quadratic Casimir of representation $r$ [15, 16, 17]. Again, it is hard to see how vortex condensation would account for this "Casimir scaling" of string tensions in the intermediate distance regime.

From these considerations, it is clear that the "Confiners," whatever they may be, must produce rather different effects in different distance regimes. In $SU(2)$ gauge theory, in the intermediate distance regime, the Confiners should supply string tensions compatible with

$$\frac{\sigma_j}{\sigma_{\frac{1}{2}}} \approx \frac{4}{3} j(j+1) \tag{17}$$

while, from the onset of color-screening and beyond, they should produce asymptotic values

$$\sigma_j = \begin{cases} \sigma_{\frac{1}{2}} & j = \text{ half-integer} \\ 0 & j = \text{ integer} \end{cases} \tag{18}$$

Both the vortex-condensation and abelian-projection theories are compatible with the latter condition on asymptotic string tensions, but do not explain Casimir scaling at intermediate distances.

It is entirely possible that $Z_N$ vortices (or, for that matter, magnetic monopole configurations) have something to do with the confinement mechanism at distance scales beyond the onset of color-screening. But it is also possible that the data of the previous section could be misleading in some way, and it is worth considering how that could happen. Let us rewrite eq. (14) in the form

$$W(C) = \Delta P(C) W_{evn}(C) + P_{odd}(C) \Delta W(C) \tag{19}$$

where

$$\begin{aligned} \Delta P(C) &\equiv P_{evn}(C) - P_{odd}(C) \\ \Delta W(C) &\equiv W_{evn}(C) + W_{odd}(C) \end{aligned} \tag{20}$$

In the center projection, $W_{evn} = 1$ and $W_{odd} = -1$ so that

$$W_{cp}(C) = \Delta P(C) \tag{21}$$

If $Z_2$ vortices are the confiners, then, as in the center projection, the area law is due to random fluctuations in the number of vortices piercing the loop. It would then be the term proportional to $\Delta P(C)$ in eq. (19) which accounts for the asymptotic string tension in the full theory. Asymptotically, $P_{evn} \approx P_{odd} \approx \frac{1}{2}$, so that

$$W(C) \rightarrow \Delta P(C) W_{evn} + \frac{1}{2} \Delta W(C) \tag{22}$$

To really establish that $Z_2$ vortices are the origin of the asymptotic string tension, we need (among other things) to establish that

$$\Delta P(C) \sim \exp[-\sigma_{cp} A(C)] \tag{23}$$

with

$$\sigma_{cp} = \sigma_{fund} \tag{24}$$

where $\sigma_{cp}$ is the string tension of the fundamental representation in center projection. Proper scaling of $\sigma_{cp}$ with respect to $\beta$ is a necessary but not sufficient condition for this equality, and this is one way that the previous data might be misleading. If it should turn out that

$$\sigma_{cp} > \sigma_{fund} \tag{25}$$

then the first term on the rhs of eq. (22) would become negligible, asymptotically, compared to $W(C)$, and the asymptotic string tension would have to be due to $\Delta W(C)$.

From Fig. 2, and the scaling apparent in Fig. 1, it would appear that $\sigma_{fund}$ and $\sigma_{cp}$ are not very different. In this connection, it is instructive to compare Fig. 2 with

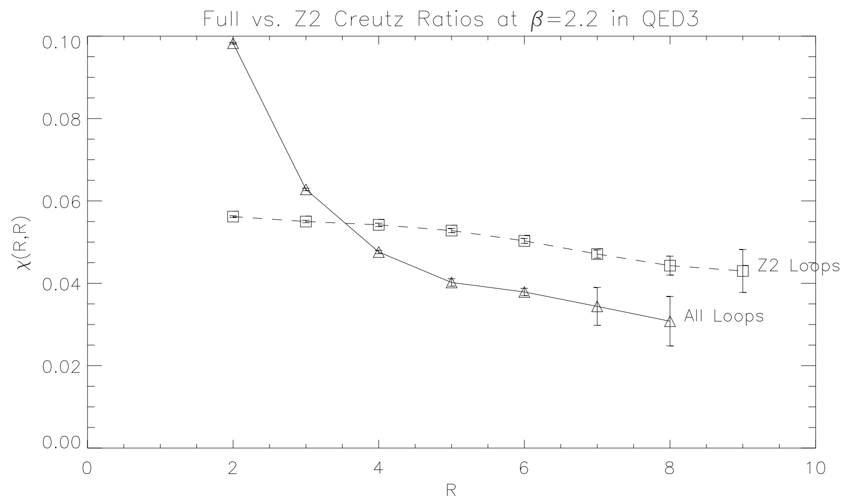


Figure 10: Test of $Z_2$ dominance in compact $QED_3$: $Z_2$ Projected vs. Full Creutz ratios at $\beta = 2.2$ on a $30^3$ lattice.

an analogous calculation in compact $QED_3$. In $QED_3$ it is also possible to define a "maximal $Z_2$ gauge," via eq. (4), and a "$Z_2$ projection" (the term "center projection" would be a misnomer here) according to eq. (5). A sample result for lattice $QED_3$ is shown below in Fig. 10. This simulation was run on a $30^3$ lattice at $\beta = 2.2$, with 5000 thermalizations followed by 10000 sweeps, with data taken every 10th sweep. In this case, Creutz ratios $\chi_{cp}(R, R)$ for the projected data appear to be approximately 40% larger than the Creutz ratios $\chi(R, R)$ for the unprojected data, and the two quantities don't appear to be converging for larger loops.

The agreement between projected and unprojected Creutz ratios appears to be substantially better in the $D = 4$ $SU(2)$ theory than in compact $QED_3$, despite the fact that $SU(2)$ is a larger group than $U(1)$. On the other hand, the data presented in section 3 for the non-abelian theory was obtained on workstations, not supercomputers. A state-of-the art string tension calculation aimed at a better quantitative comparison of $\sigma$ and $\sigma_{cp}$, and perhaps also a study of alternate versions of the maximal center gauge (such as (7)), is certainly called for.

# 5 Conclusions

None of the evidence gathered so far is conclusive, although it does seem to point in a certain direction. We have found that:

1. Center-projected link variables have the property of "center dominance" in maximal center gauge. In this gauge it is the sign alone, of the real part of the abelian-projected link, which appears to carry most of the information about the asymptotic string tension.

2. Vortices in the center-projected configurations ("P-vortices") appear to be strongly correlated with the presence or absence of confining field configurations in the full, unprojected field configurations. When Wilson loops are evaluated in an ensemble of configurations which do not contain P-vortices within the loop, the asymptotic string tension disappears.

3. Wilson loops $W_0(C)$, $W_1(C)$, $W_2(C)$, evaluated in ensembles of configurations containing respectively zero, one, or two P-vortices inside the loop, in the corresponding center projection, behave as though they contained zero, one or two $Z_2$ vortices in the full, unprojected configuration. That is, $W_1/W_0 \rightarrow -1$, and $W_2/W_0 \rightarrow +1$, as the loop area increases.

If the Yang-Mills vacuum is dominated by $Z_2$ vortices, as this data would seem to suggest, it raises many puzzling questions. Foremost among these is how to account for the existence and Casimir scaling of the adjoint string tension. Because of the existence of the adjoint tension, we think it unlikely that fluctuations in the number and location of $Z_2$ vortices can give a complete account of the confinement mechanism in the intermediate distance regime. Such vortex fluctuations *could* be decisive asymptotically; further work will be needed to find out.

Perhaps the most urgent need is to repeat all of the calculations reported here for the case of an $SU(3)$ gauge group. If there is center dominance (with $\sigma \approx \sigma_{cp}$), and if the presence of P-vortices is correlated with the magnitude of the string tension, and especially if

$$\frac{W_1(C)}{W_0(C)} \quad \longrightarrow \quad e^{2\pi i/3} \tag{26}$$

for corresponding P-fluxons with one unit $e^{2\pi i/3}$ of center flux, then we believe that the combined evidence in favor of some version of the $Z_N$ vortex condensation theory would become rather compelling.

## Acknowledgements

J.G. is grateful for the hospitality of the Niels Bohr Institute, where some of this work was carried out. He would also like to thank J. Ambjørn and T. H. Hansson for discussions. This research was supported in part by an EC HMC Institutional Fellowship under Contract No. ERBCHBGCT930470 (L.DD.), the Hochschuljubiläumsstiftung der Stadt Wien H-00114/95 (M.F.), the U.S. Dept. of Energy, under Grant No. DE-FG03-92ER40711 (J.G.), and the Slovak Grant Agency for Science, Grant No. 2/1157/94 (Š.O.). Support was also provided by the Director, Office of Energy Research, Office of Basic Energy Services, of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

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